# A Reconfiguration Control Strategy using TKS Model Predictive Control for Network Control Systems. A Magnetic Levitation Case Study.

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Abstract: This paper presents a reconfiguration control strategy for Network Control Systems that makes use of a Fuzzy Takagi-Sugeno Model Predictive Control. The dynamic behaviour of a Network Control System is modelled by using a real-time implementation of the scheduling algorithm. Here, this is applied for a magnetic levitation system, as a plant that is also modelled using a Fuzzy Takagi-Sugeno approach. Thus, this paper covers several design issues, like for instance, how to model a computer network, a plant, and a reconfiguration control strategy, as well as how the reconfiguration control strategy is modified using the Fuzzy approach.

### 1. Introduction

Reconfiguration is a transition that modifies the structure of a system so it changes its representation of states. Here, it is used as a feasible approach for fault isolation, and also, it is a response to time delay modification.

In control systems, several modelling strategies for managing time delay within control laws have been studied by different research groups. Nilsson (1998) proposes the use of a time delay scheme integrated to a reconfigurable control strategy, based on a stochastic methodology. Jiang et al., (1999) describe how time delays are used as uncertainties, which modify pole placement of a robust control law. Izadi et al., (1999) present an interesting case of fault tolerant control approach related to time delay coupling. Blanke et al. (2003) study reconfigurable control from the point of view of structural modification, establishing a logical relation between dynamic variables and the respective faults. Finally, Thompson (2004) and Benítez-Pérez et al. (2005) consider that reconfigurable control strategies perform a combined modification of system structure and dynamic response, and thus, this approach has the advantage of bounded modifications over system response.

Normally, when a fault occurs during the operation of a system, a respective fault tolerance strategy is applied.

However, applying such a fault tolerance strategy is not enough to maintain the performance of the system, since dynamic conditions are modified. Therefore, it is seems necessary to take into account current conditions in order to keep system performance, even degraded. Thus, this paper proposes a novel technique based on Fuzzy MPC control and considering bounded variable time delays. Here, local faults and inherent time delays are used as necessary conditions for control design.

The approach here makes use of a case study that takes time delays due to communication as deterministic measured variables: a light sensor, for which a fault is established as a deviation bigger than 50% of the current value, and the related time delays are used for modelling the control law. For this, a Fuzzy MPC law (Abonyi, 2003) is used, where time delays result from the deterministic reconfiguration of communications due to a scheduling algorithm. MPC is used for managing extended horizons from system inputs and outputs, to determine several scenarios modified by time delays. Recent results encourage this approximation, as shown in Benitez-Perez (2008a, 2008b).

For experimental purposes, the following considerations are taken:

- 1. Faults are strictly local in peripheral elements, and these are tackled by just eliminating the faulty element. In fact, faults are catastrophic and local.
- 2. Time delays are bounded and restrictive to scheduling algorithms.
- 3. Global stability is reached by using a classical control strategy.
- 4. For each fault scenario, the combination tends to be globally stable, although a Fuzzy TKS is used.

The objective of this paper is to present a reconfiguration control strategy developed from the time delay knowledge, as well as local fault effects within a distributed system environment, for a magnetic levitation case study. The novelty is to propose a Takagi-Sugeno (TKS) Model Predictive Control (MPC) for Network Control System (NCS) based on the defined reconfiguration.

### 2. Structural Reconfiguration Algorithm

This paper focuses on the reconfiguration of the control law due to local faults and consequent time delays, as shown in Figure 1. Time delays are measurable and bounded, according to a real-time scheduling algorithm. Here, the scheduling algorithm is the EDF algorithm (Liu, 2002).

From Figure 1, it is noticeable that structural reconfiguration takes place as a result of EDF, which makes use of an ART2A neural network (García-Zavala et al. 2005), whose action causes a control law transition. The aim here is to study how this transition is carried out when using a Fuzzy TKS approach (Abonyi, 2003), based on MPC.



Figure 1. General structure of Reconfigurable System over a Computer Network

The core of the structural reconfiguration algorithm is to perform an on-line reconfiguration by using an ART2 neural network (Frank et al. 1998) to classify valid and non-valid plans:

- First, the ART2 is trained off-line, using valid and non-valid plans from EDF evaluation and case study response.
- Based on this training procedure, two main types of reconfigurations are determined: suitable reconfigurations and non-trustable reconfigurations.
- During on-line stage, ART2 allows classification from new plans.
- If the response of ART2 belongs represents a valid plan, the reconfiguration is performed; otherwise the proposed plan is rejected.
- An important constraint is that ART2 cannot learn new plans during on-line stage, as a safety precaution.

For a NCS, the communication network strongly affects the dynamics of the system, expressed as a time variance that exposes a nonlinear behaviour. Such nonlinearity is addressed by incorporating time delays. From real-time system theory, it is known that time delays are bounded even in the case of causal modifications due to external effects. Using this representation, time delays are counted using simple addition, as described in the next section.

## 3. Case Study and Reconfiguration Approach

The case study here is a magnetic system integrated to a computer network as shown in Figure 2 (Wincon, 2003).



Figure 2. Magnetic Levitator Case Study

The dynamics of case study are expressed in terms of a transfer function as:

$$G_{bi}(s) = \frac{-k_{bdc} w_b^2}{s^2 - w_b^2}$$

$$k_{bdc} = \frac{x_{bo}}{I_{co}}$$

$$w_b = \sqrt{\frac{2g}{x_{bo}}}$$
(1)

where:

- *g* is the force of gravity,
- $I_{co}$  is the current in the coil, and
- $x_{bo}$  is the distance from coil to the ball position.

For experimental purposes, Figures 3 and 4 present time diagrams respectively for a fault free scenario and a fault scenario considering fault masking. In these figures,  $s_1$  and  $s_2$  are optic sensor nodes, C is the control node, and  $A_1$  is the actuator node. When a fault occurs, EDF is used, and the ART2A re-organises task execution according to time restrictions. Notice that in Figures 3 and 4 the maximum time delays are bounded.





Figure 4. Fault scenario considering fault masking

Both scenarios are local with respect to magnetic levitation system: they are not periodic, although nodes are periodic. As both scenarios are bounded, the consumption times are expressed as Equations 3 and 4 (from Figures. 3 and 4, respectively). For fault free scenario, time delay is expressed as:

$$T = 4t_s + t_{cm}^{sc} + t_c + t_{cm}^{ca} + t_a$$
(3)

while for fault scenario, time delay is expressed as:

$$T = 4t_s + t_{cm}^{sc} + t_{fm}^{sft} + t_{ft} + t_{cm}^{fsc} + t_c + t_{cm}^{ca} + t_a$$
(4)

#### where:

- *t*<sub>s</sub> is the time consumed by sensors,
- $t_{cm}^{sc}$  is the communication time between sensor and control,
- $t_{cm}^{sft}$  is the communication time between sensor and fault tolerance module,
- $t_{ft}$  is the consumption time from fault tolerance module,
- $t_{cm}^{fsc}$  is the time consumed for the fault sensor to send messages to its neighbour and produce agreement,
- $t_c$  is the time consumed by control node,
- $t_{cm}^{ca}$  is the communication time between controller and actuator, and
- $t_a$  is the time consumed by actuator.

From both scenarios, it is defined a *fault tolerance* element that represents some extra communication for control performance, although it masks any local fault from sensors. From this time boundary, in both scenarios, it is feasible to implement some control strategies. Considering this, two possible fault cases are feasible:

- One local fault;
- Several local faults.

From these two possible fault cases, the latter is a worst-case scenario, related to several local faults that have an impact on the global control strategy. However, the first case presents a minor degradation on the global control strategy. Despite this degradation, the system is expected to keep its functionality, given the fault tolerance strategy and the local time delays integrated into related controllers. Faults are local failures of light capture on each optic sensor. The fault tolerant strategy is performed within the fault tolerance element. Local faults are related to local values deviations from current measures. Moreover, such local faults are neither catastrophic, nor permanent. Considering the two possible fault cases, Table 1 contains the measured values for local and global time delays.

Configuration 1	Local Time Delays	1 ms
Several Local Fault	Global Time Delays	5 ms
Configuration 2	Local Time Delays	1 ms

Table 1. Time delays related to local communications

As the time delays are bounded, the plant model is defined as an integration of the original plant and the control, this is,

from Equations 1 and 2 (Figure 5).



Fig. 5. Plant and control integration

The plant model has the following dynamics:

$$\begin{aligned} x(k+1) &= a^p x(k) + B^p u(k) \\ y &= c^p x(k) \end{aligned} \tag{5}$$

where:

- $a^p \in \Re^{nxn}$ ,
- $c^p \in \mathfrak{R}^{nx_1}$
- $B^p \in \Re^{nx1}$  are matrices related to the plant, and
- x(k), u(k) and y(k) are states, inputs, and outputs, respectively.

In particular,  $B^{P}$  is defined as:

$$B^{p} = \sum_{i=1}^{N} \rho_{i} B_{i} \sum_{j=1}^{M} \int e^{-a^{p} \left(t-\tau\right)} d\tau$$
(6)

where:

• 
$$\rho_i = \{0,1\}, \sum_{i=1}^N \rho_i = 1$$

- *N* is the total number of possible faults,
- *M* is the involved time delays from each fault,
- $\tau_{j-1}^i$  and  $\tau_j^i$  are current communication time delay,  $\sum_{j=1}^M \tau_j^i \leq T$  where **T** is the total transport delay of the

cycle and depends of the faults scenarios.

Thus,  $B_i$  is an array:

$$B_i = \begin{bmatrix} b_1 \\ b_2 \\ 0_i \\ \vdots \end{bmatrix}$$

where:

- $b_1 \rightarrow b_N$  are the elements conformed at the input of the plant (such as actuators), and
- $0_i$  is the lost element due to local actuator fault.

 $B^{p}$  represents only one scenario (Equation 6). A further definition of a current  $B_{i}^{p}$  considers local actuator faults and related time delays:

$$B_{i}^{p} = B_{i} \sum_{j=1}^{M} \int_{\tau_{j}^{j}}^{\tau_{j-1}^{j}} e^{-a^{p} \left(t-\tau\right)} d\tau$$
(7)

For simplicity,  $B_i^p$  is used in order to describe local linear plants.

From this representation, a fuzzy plant is defined as follows, taking into consideration each time delay, fault cases and the related fuzzy rules:

$$r_i$$
: if  $x_1$  is  $\mu_{1i}$  and  $x_2$  is  $\mu_{2i}$  and ... and  $x_l$  is  $\mu_{li}$  then  $a_i^p x(k) + B_i^p u(k)$  (8)

where:

- $\{x_1x_2...,x_1\}$  are current state measures,
- *l* is the number of states,
- $i = \{1, ..., N\}$  is one of the fuzzy rules,
- N is the number of the rules which is equal to the number of possible faults, and
- $\mu_{ij}$  are the related membership functions, which are Gaussian defined as:

$$\mu_{ij}(y_{\alpha}, u_{\alpha}) = \exp\left(-\left(\frac{y_{\alpha} - c_{ij}}{\sigma_{ij}^{y}}\right)^{2} - \left(\frac{u_{\alpha} - c_{ij}}{\sigma_{ij}^{u}}\right)^{2}\right)$$
(9)

where  $c_{ij}$  and  $\sigma_{ij}$  are parameters to be tuned.

The representation of the plant as an integrated system with the control is thus based on centre of area de-fuzzification method (Driankov, 1994). From this representation of a global nonlinear system, it is necessary to define a global stability condition as a result of this fuzzy system. This is given considering fuzzy logic control approach. The result allows the integration of nonlinear stages and transitions to basically a group of linear plants. As from the point of view of the approach, taking the input of the plant as consequent, this is defined as Fuzzy MPC as follows,:

$$u_i = (S_i^T \overline{Q} S_i + \overline{R})^{-1} S_i \overline{Q} (w - p_i)$$
<sup>(10)</sup>

where:

- *w* are the future set points,
- $u_i$  is the control output,
- *Q* and *R* are positive definite weight matrices defined as:

$$\overline{Q} = Diag(Q)$$
  
 $\overline{R} = Diag(R)$ 

• *S* represents the effect of future outputs and from the integration to antecedent representation of the fuzzy logic system (Equation 8), over *N<sub>p</sub>* and *N<sub>c</sub>* horizons defined by the user:

$$S = \begin{bmatrix} S_{Np_{I}} & S_{Np_{I}-1} & \cdots & 0 \\ S_{Np_{I}+1} & S_{Np_{I}} & S_{Np_{I}-1} \\ \vdots & \vdots & \ddots & \\ S_{Np_{2}} & S_{Np_{2}-1} & \cdots & S_{Np_{2}-N_{c}} \end{bmatrix},$$
(11)

in which:

$$s_{j} = y_{0} \qquad \forall j \le n_{d},$$

$$s_{j} = \sum_{i=1}^{Na} a_{i} S_{j-i} + \sum_{i=1}^{Nb} B^{p_{i}} \qquad j > n_{d}$$

$$p_{i} = \sum_{j=1}^{Na} a_{j} p_{i-j} + \sum_{j=1}^{Nb} b_{j} B^{p}_{j} u (k - j - n_{d} + i) + c$$
(12)

Figure 6 show how these horizons take place in time.



Figure 6 Time horizons with respect to time delays. Horizon samples and k sampling time.

In Figure 6,  $N_a$  and  $N_b$  are the horizon samples, k is the sampling time, l is the related time delay within the sampling time,  $n_d$  is the minimum discrete dead-time. In Equation 12, the parameters of the plant are presented as  $a_j$  and  $B_i^p$  where:

$$b_j = f\left(B_j^p\right)$$

and:

$$B_i^p = \int_{t_i}^{t_{i+1}} e^{(\Delta \Delta - \tau)} d\tau B$$

from the integration to antecedent representation of Fuzzy system (Equation 8):

$$D_{i}(y_{\alpha}, u_{\alpha'}) = \prod_{j=1}^{N_{p}} \mu_{ij}(y_{\alpha}, u_{\alpha'}) = \prod_{j=1}^{N_{p}} \exp\left(-\left(\frac{y_{\alpha} - c_{ij}}{\sigma_{ij}^{y}}\right)^{2} - \left(\frac{u_{\alpha'} - c_{ij}}{\sigma_{ij}^{u}}\right)^{2}\right)$$

$$\alpha = \begin{cases} 0 \quad j > N_{p} \\ j \quad 1 \le j \le N_{p} \end{cases}$$

$$\alpha' = \begin{cases} 0 \quad 1 \le j \le N_{p} \\ j \quad j > N_{p} \end{cases}$$
(13)

where:

- $N_p$  is the number of possible inputs for the fuzzy plant,
- *y* is the output of the plant, and
- *u* is the plant input.

For the antecedent part of fuzzy control  $\Omega_i$ 

$$\Omega_{i}(y_{\alpha}, u_{\alpha'}) = \prod_{j=1}^{N_{A}} \mu_{ij}(y_{\alpha}, u_{\alpha'}) = \prod_{j=1}^{N_{A}} \exp\left(-\left(\frac{y_{\alpha} - c_{ij}}{\sigma_{ij}^{y}}\right)^{2} - \left(\frac{u_{\alpha'} - c_{ij}}{\sigma_{ij}^{u}}\right)^{2}\right)$$

$$\alpha \begin{cases} 0 \quad j > N_{A} \\ j \quad 1 \le j \le N_{A} \end{cases}$$

$$\alpha' \begin{cases} 0 \quad 1 \le j \le N_{A} \\ j \quad j > N_{A} \end{cases}$$
(14)

where  $N_A$  is the number of possible inputs for the fuzzy controller, following that expressed in Figure 6.

In this case, fault conditions are presented as the results of local time delays more than the actual loss of current measure. Remember that Equations 3 and 4 are the basis for time delay by using EDF as scheduling algorithm, presented as:

$$4t_{s} + t_{cm}^{sc} + t_{c} + t_{cm}^{ca} + t_{a} < T$$

Thus, the plant representation is given by Equation 15 considering time delays:

$$x(k+1) = \frac{\sum_{i=1}^{N} D_i(y_{\alpha}, u_{\alpha'}) (a_i x(k) + B_i^p u(k))}{\sum_{i=1}^{N} D_i(y_{\alpha}, u_{\alpha'})}$$
(15)

where N is the number of rules, and the plant input is defined as considering time delays expressed in Equation 14:

$$u(k) = \frac{\sum_{i=1}^{N} \Omega_i(y_{\alpha}, u_{\alpha'}) \left( S_k^T \overline{Q} S_k + R \right)^{-1} S_k \overline{Q} (w - p_k)}{\sum_{i=1}^{N} \Omega_i(y_{\alpha}, u_{\alpha'})}$$
(16)

Substituting in Equation 15:

$$x(k+1) = \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'}) \left( a_{i} x(k) + B_{i}^{p} \frac{\sum_{i=1}^{N} \Omega_{i}(y_{\alpha}, u_{\alpha'}) \left( S_{k}^{T} \overline{Q} S_{k} + \overline{R} \right)^{-1} S_{k} \overline{Q} (w - p_{k}) \right)}{\sum_{i=1}^{N} \Omega_{i}(y_{\alpha}, u_{\alpha'})}$$
(17)

On the other hand, in order to establish valid horizonts considering time delays and failures MPC strategy is used, therefore, the cost function in MPC is defined as

$$J = \sum_{i=1}^{N_p} B_i^p (ref_i - y_i)^2 + \sum_{i=1}^{N_A} \delta_i (u_i)^2$$
(18)

where  $ref_i$  and  $y_i$  are the reference and output values respectively. This equation can be rewritten as:

$$J = \sum_{k=1}^{N_p} B_k^p \left( ref_k - Cx(k-1) \right)^2 + \sum_{k=1}^{N_A} \delta_k \left( u_k \right)^2$$
(19)

Considering the variables x and  $u_i$  defined in Equations 15 and 16:

$$J = \sum_{k=1}^{N_{p}} B_{k}^{p} \left( ref_{k} - C \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'}) (a_{i}x(k-2) + B_{i}^{p}u(k-2))}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'})} \right)^{2} + \sum_{k=1}^{N_{A}} \delta_{k} \left( \frac{\sum_{i=1}^{N} \Omega_{i}(y_{\alpha}, u_{\alpha'}) (S_{k}^{T} \overline{Q} S_{k} + \overline{R})^{-1} S_{k} \overline{Q}(w - p_{k})}{\sum_{i=1}^{N} \Omega_{i}(y_{\alpha}, u_{\alpha'})} \right)^{2}$$
(20)

Since the values of Q, S and R are defined as positive definite matrices in Equation 10, it is necessary to obtain the partial derivatives for each variable in order to get the optimal values as:

$$\frac{\partial J}{\partial \mathbf{B}_{k}^{p}} = 2\sum_{k=1}^{N_{p}} B_{k}^{p} \left( ref_{k} - Cx(k-1) \right)$$
(21)

$$\frac{\partial J}{\partial \delta_k} = 2 \sum_{k=1}^{N_A} \delta_k(u_k)$$
(22)

$$\frac{\partial J}{\partial a_{i}} = 2C \sum_{k=1}^{N_{p}} B_{k}^{p} \left( ref_{k} - C \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) (a_{i}x(k-2) + B_{i}^{p}u(k-2))}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha^{i}}) x(k-2)} \right) \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha^{i}}) x(k-2)} \right) \right) \left( \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha^{i}}) x(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha^{i}}) x(k-2)$$

(23)

$$\frac{\partial J}{\partial B_{i}^{p}} = 2\sum_{k=1}^{N_{p}} B_{k}^{p} \left( ref_{k} - C \frac{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}}) (a_{i}x(k-2) + B_{i}^{p}u(k-2))}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right) \left( \frac{Cu(k-2)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha^{i}})} \right)$$
(24)

$$\frac{\partial J}{\partial D_{i}} = 2C \sum_{k=1}^{N_{p}} B_{k}^{p} \left( ref_{k} - Cx(k-1) \right) \left( \frac{\sum_{i=1}^{N} \left( a_{i} x(k-2) + B_{i}^{p} u(k-2) \right) - Nx(k-1)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'})} \right)$$
(25)

$$\frac{\partial J}{\partial \Omega_i} = 2\sum_{k=1}^{N_A} \delta_k u(k) \left( \frac{\sum_{i=1}^N \left( S_k^T \overline{\mathcal{Q}} S_k + \overline{R} \right)^{-1} S_k \overline{\mathcal{Q}} (w - p_k) - N u(k)}{\sum_{i=1}^N \Omega_i (y_\alpha, u_{\alpha'})} \right)$$
(26)

Using Equation 13 to obtain the partial derivatives of  $D_i$  with respect to  $c_{ij}$  and  $\sigma_{ij}$ :

$$\frac{\partial D_{i}}{\partial c_{ij}^{y}} = -2\sum_{k=1}^{N_{p}} \left( \frac{y_{\alpha} - c_{ik}^{y}}{\left(\sigma_{ik}^{y}\right)^{2}} \right) \exp\left( -\left( \frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ik}^{y}} \right)^{2} \right) \prod_{j=1, j \neq k}^{N_{p}} \exp\left( -\left( \frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}} \right)^{2} - \left( \frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}} \right)^{2} \right) \right)$$
(27)

$$\frac{\partial D_{i}}{\partial c_{ij}^{u}} = -2\sum_{k=1}^{N_{p}} \left( \frac{u_{\alpha'} - c_{ik}^{u}}{\left(\sigma_{ik}^{u}\right)^{2}} \right) \exp\left( -\left( \frac{u_{\alpha'} - c_{ik}^{u}}{\sigma_{ik}^{u}} \right)^{2} \right) \prod_{j=1, j \neq k}^{N_{p}} \exp\left( -\left( \frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}} \right)^{2} - \left( \frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}} \right)^{2} \right) \right)$$
(28)

Analogously, for  $\sigma^{y}_{ij}$  and  $\sigma^{u}_{ij}$ :

$$\frac{\partial D_{i}}{\partial \sigma_{ij}^{y}} = -\frac{1}{2} \sum_{k=1}^{N_{p}} \frac{\left(y_{\alpha} - c_{ik}^{y}\right)^{2}}{\left(\sigma_{ik}^{y}\right)^{3}} \exp\left(-\left(\frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ik}^{y}}\right)^{2}\right) \prod_{j=1, \ j \neq k}^{N_{p}} \exp\left(-\left(\frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}}\right)^{2} - \left(\frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}}\right)^{2}\right)$$
(29)

$$\frac{\partial D_{i}}{\partial \sigma_{ij}^{u}} = -\frac{1}{2} \sum_{k=1}^{N_{p}} \frac{\left(u_{\alpha'} - c_{ik}^{u}\right)^{2}}{\left(\sigma_{ik}^{u}\right)^{3}} \exp\left(-\left(\frac{u_{\alpha'} - c_{ik}^{u}}{\sigma_{ik}^{u}}\right)^{2}\right) \prod_{j=1, \ j \neq k}^{N_{p}} \exp\left(-\left(\frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}}\right)^{2} - \left(\frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}}\right)^{2}\right)$$
(30)

Applying the previous results, and using Equation 25:

$$\frac{\partial J}{\partial c_{ij}^{y}} = \frac{\partial J}{\partial D_{i}} \frac{\partial D_{i}}{\partial c_{ij}^{y}} = 2C \sum_{k=1}^{N_{p}} B_{k}^{p} \left( ref_{k} - Cx(k-1) \right) \left( \frac{\sum_{i=1}^{N} \left( a_{i}x(k-2) + B_{i}^{p}u(k-2) \right) - Nx(k-1)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'})} \right) \times \left( -2\sum_{k=1}^{N_{p}} \left( \frac{y_{\alpha} - c_{ik}^{y}}{\left(\sigma_{ik}^{y}\right)^{2}} \right) \exp\left( -\left( \frac{y_{\alpha} - c_{ik}^{y}}{\sigma_{ik}^{y}} \right)^{2} \right) \prod_{j=1, \ j \neq k}^{N_{p}} \exp\left( -\left( \frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}} \right)^{2} - \left( \frac{u_{\alpha'} - c_{ij}^{y}}{\sigma_{ij}^{y}} \right)^{2} \right) \right) \right)$$
(31)

and applying the same for  $\ensuremath{\mathcal{C}}^u_{ij}$  :

$$\frac{\partial J}{\partial c_{ij}^{u}} = \frac{\partial J}{\partial D_{i}} \frac{\partial D_{i}}{\partial c_{ij}^{u}} = 2C \sum_{k=1}^{N_{p}} B_{k}^{p} \left( ref_{k} - Cx(k-1) \right) \left( \frac{\sum_{i=1}^{N} \left( a_{i}x(k-2) + B_{i}^{p}u(k-2) \right) - Nx(k-1)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'})} \right) \times \left( -2\sum_{k=1}^{N_{p}} \left( \frac{u_{\alpha'} - c_{ik}^{u}}{\left(\sigma_{ik}^{u}\right)^{2}} \right) \exp\left( -\left( \frac{u_{\alpha'} - c_{ik}^{u}}{\sigma_{ik}^{u}} \right)^{2} \right) \prod_{j=1, j \neq k}^{N_{p}} \exp\left( -\left( \frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}} \right)^{2} - \left( \frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}} \right)^{2} \right) \right) \right)$$
(32)

Analogously for  $\sigma^{\mathrm{y}}_{\mathrm{ij}}$  and  $\sigma^{\mathrm{u}}_{\mathrm{ij}}$  :

$$\frac{\partial J}{\partial \sigma_{ij}^{y}} = \frac{\partial J}{\partial D_{i}} \frac{\partial D_{i}}{\partial \sigma_{ij}^{y}} = 2C \sum_{k=1}^{N_{p}} B_{k}^{p} (ref_{k} - Cx(k-1)) \left( \frac{\sum_{i=1}^{N} (a_{i}x(k-2) + B_{i}^{p}u(k-2)) - Nx(k-1)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'})} \right) \times \left( -\frac{1}{2} \sum_{k=1}^{N_{p}} \frac{(y_{\alpha} - c_{ik}^{y})^{2}}{(\sigma_{ik}^{y})^{3}} \exp\left( -\left(\frac{y_{\alpha} - c_{ik}^{y}}{\sigma_{ik}^{y}}\right)^{2}\right) \prod_{j=1, j \neq k}^{N_{p}} \exp\left( -\left(\frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}}\right)^{2} - \left(\frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}}\right)^{2}\right) \right)$$
(33)

$$\frac{\partial J}{\partial \sigma_{ij}^{u}} = \frac{\partial J}{\partial D_{i}} \frac{\partial D_{i}}{\partial \sigma_{ij}^{u}} = 2C \sum_{k=1}^{N_{p}} B_{k}^{p} (ref_{k} - Cx(k-1)) \left( \frac{\sum_{i=1}^{N} (a_{i}x(k-2) + B_{i}^{p}u(k-2)) - Nx(k-1)}{\sum_{i=1}^{N} D_{i}(y_{\alpha}, u_{\alpha'})} \right) \times \left( -\frac{1}{2} \sum_{k=1}^{N_{p}} \frac{(u_{\alpha'} - c_{ik}^{u})^{2}}{(\sigma_{ik}^{u})^{3}} exp \left( -\left( \frac{u_{\alpha'} - c_{ik}^{u}}{\sigma_{ik}^{u}} \right)^{2} \right) \prod_{j=1, j \neq k}^{N_{p}} exp \left( -\left( \frac{y_{\alpha} - c_{ij}^{y}}{\sigma_{ij}^{y}} \right)^{2} - \left( \frac{u_{\alpha'} - c_{ij}^{u}}{\sigma_{ij}^{u}} \right)^{2} \right) \right)$$
(34)

A similar procedure is used to obtain the partial derivatives with respect to  $c_{ij}$  and  $\sigma_{ij}$  using Equation 14. The optimization procedure of  $\delta$ , Q, and R are left to the use of this multivariable optimization procedure, since these are design variables.

## 5. Results

Using this implementation, several experiments are carried out, whose results are presented in terms of fault presence and its related fault tolerance strategy to overcome system lack of performance regarding an horizon. How the system responds to the fault tolerance strategy is presented as follows, showing the error response from different separation values between membership functions, according to Table 2.

Separation(%)	Integral of the error		
10	0.4400		
15	0.4495		
20	0.4635		
25	0.4642		
30	0.5637		
40	0.8491		
50	0.7498		

Table 2. Fault-free scenario for different percent of separation values between membership functions.

Figure 7 shows fault-free scenarios for 10% and 15% of this separation between membership functions, as presented in Benitez-Perez et al. (2008b). Faults refer to light capture that detects current movement of the ball of the magnetic levitation case study. Both faults are local deviations of current responses from light sensors. These are not catastrophic

faults, but only partial deviation from current measurements. These faults are presented locally, and within a time frame bigger than the sampling time.



Figure 7. Error response from fault-free scenario

For both fault scenarios, the response of the systems is shown in Table 3.

Separation(%)	Integral of the error
10	0.5003
15	0.4567

Table 3. The integral of the error for fault scenario.

Figure 8 shows the error response for each fault scenario, when switching from one sensor to another, using the separation of 10% and 15%.



Figure 8. Error response from fault scenario

Figure 9 presents the response of the system compared with the current set point.



Figure 9. Set point and System Response from Case Study

This last example presents the reconfiguration control strategy based on the decision-maker module and the related Fuzzy MPC. Active switching control is performed using a Fuzzy TKS approach when a local fault appears, and fault tolerant node is activated. Such a reconfiguration control strategy is feasible due to the knowledge about time delays. Notice that the consumption time of the reconfiguration control strategy is neglected, since it is considered part of control performance. It is obvious that fault presence is measurable: if a local fault cannot be detected, the strategy becomes useless. Alternatively, local time delay management refers to the use of a quasi-dynamic scheduler to propose dynamic reconfiguration, based on current system behaviour rather than predefined scenarios.

To define the communication network performance, the use of **xPC Target** is necessary. Such a network implementation uses message transactions that are implemented in the real-time workshop toolbox from MATLAB (Hanselman et al., 2002). For the case study, two types of computer network are used: CANbus and Ethernet. Both networks present no further time delay difference because network size is kept quite small.

### 6. Concluding Remarks

The present paper presents an approach for the integration of two techniques in order to perform reconfiguration: structural reconfiguration and control reconfiguration. These two techniques are applied in cascade. Although there is no formal verification for following this sequence, it has been adopted since structural reconfiguration provides stable conditions for control reconfiguration. Moreover, the use of a real-time scheduling algorithm to approve or disapprove changes on the behaviour of a computer network allows bounding time delays during a specific time frame. This local time delay allows the design of a control law, capable to cope with new conditions.

Preliminary results show that the proposed reconfiguration control strategy is feasible as long as the use of a wide enough horizon predetermines which control law is adequate. This is accomplished by the composition ed of two algorithms: one responsible for structural reconfiguration (and implemented here as ART2A), and the second responsible control reconfiguration (here based on for Fuzzy TKS and MPC). The importance of this approach is that control conditions are strictly bounded to certain response. Future work is related to integrate dynamic scheduling algorithms and formal stability probe for this implementation.

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