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Frequency Transition for scheduling management using dynamic system approximation for a kind of NCS

O. Esquivel-Flores[1], H. Benítez-Pérez[3],
P. E. Mendez-Monroy[2], Antonio Menéndez[3]

[1]Posgrado en Ciencia e Ingeniería de la Computación, Universidad Nacional Autónoma de México, México D.F. (e-mail: oaefmcc@hotmail.com)

[1]Programa de Maestría y Doctorado en Ingeniería, Campo Ingeniería Eléctrica, Universidad Nacional Autónoma de México, México D.F. (e-mail: oaefmcc@hotmail.com)

[2]Departamento de Ingeniería de Sistemas Computacionales y Automatización
Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México
Apdo. Postal 20-726. Del. A. Obregón, México D.F.
C.P. 01000, México. Fax:++52.55.56.16.01.76,Tel:++52.55.56.22.32.12
(e-mail: hector@uxdea4.iimas.unam.mx)

ABSTRACT. *This paper provides a novel approach to schedule a real time distributed system through modifications on frequency transmission of individual components in the system. In this work we propose a dynamic linear time invariant model based upon relationship between frequencies of agents in a distributed system and using LQR control scheme to bring the system into a nonlinear region. Numerical simulations show the model effectiveness using as case of study a prototype of helicopter.*
Keywords: Distributed Systems, Frequency, Scheduling, LQR Control, Agents

1. Introduction

At the present time distributed systems are widely used in the industrial an research. These systems fulfills critical mission and long-running applications, some characteristics of the distributed computation systems are either capacity to maintain consistency or recovering without suspending their execution. These systems should complete time restrictions, coherence, adaptability, stability among others. In order to achieve overall objectives of all tasks performed, it is necessary for all agents to exchange their own information through communication media properly. Therefore the mechanism of communicating information plays an important role on stability and performance of the control systems implemented over communication network [1].

The network scheduling is a priority in the design of a NCS, when a group of agents are linked together through the available network resources. If there is no coordination among agents, data transmissions may occur simultaneously and it has to back off to avoid collisions or bandwidth violations. This results in transmission of some real time with delay or even failure to comply their deadlines. A good scheduling control algorithm tries to minimize this loss of system performance [2]. Nevertheless there isn't a global scheduler that guarantees an optimal system performance [3].

Mainly due to communication network introduce a number of issues not deal properly, Lian et al. [4],[5] have designed methodologies for networked nodes (agents of the system) to generate proper control actions and utilize communication bandwidth optimally. The effectiveness of the digital control system depends on the sampling rate. A region which control performance is acceptable deals with two points α and β associated to α_w and β_w sampling rates which can be determined by characteristics an

statistics of networked induced delays and device processing time delays. implies that small sampling periods could be necessary to guarantee a certain level of control performance, as the sampling rate gets faster , the network traffic load becomes heavier, the possibility of more contention time or data loss increase in a bandwidth-limited network and longer time delays result. Hence it's very important to considerate either sampling periods or frequency transmission to obtain better system performance. On other hand, the basic idea of a consensus algorithm is to impose similar dynamics on the information states of each agent involved in a dynamical system [6]. In networks of agents consensus means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents. Recently several problems related to multi-agent networked systems with close ties to consensus problems have got an important interest. Olfati-Saber et al.in [7] present an overview of the key results on theory and applications of consensus problems in networked systems which include control theoretic methods for convergence and performance analysis of consensus protocols. This paper aims to show that scheduling of a distributed system can be accomplished through the modification of transmission frequencies which are controlled so that their values remain in a region where the system performance is not affected. The authors propose a lineal time invariant model in which the coefficients of the state matrix are the relations between the frequencies of each node and it use a LQR feedback controller that modifies transmission frequencies bounded between maximum and minimum values of transmission in which ensures the system's schedulability.

The rest of this paper is organized as follow, in the section 2 we show a frequency transition model and a proposal to matrix coefficients of the model, in section 3 we present a dynamical system of a helicopter prototype. In section 4 we present numerical simulations of the model presented and show the performance of LQR controller. Finally, we conclude in section 5.

2. Frequency transition model

Let a distributed system with n nodes or agents that perform one task t_i with period p_i and consumption c_i each one for $i = 1,2,...,n$. The distributed system dynamics can be modeled as a linear time-invariant system which state variables $x_1, x_2, ..., x_n$ are the frequencies of transmission $f_i = \frac{1}{p_i}$ of n nodes involved on it. The authors assume there is a relationship between frequencies $f_1, f_2, ..., f_n$ and external input frequencies $u_1, u_2, ..., u_n$ which serve as coefficients of the linear system:

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{1}$$

$A \in \Re^{nxn}$ is the matrix of relationships between frequencies of the nodes, $B \in \Re^{nxn}$ is the scale frequencies matrix, $C \in \Re^{nxn}$ is the matrix with frequencies ordered, $x \in \Re^n$ is a real frequencies vector, $y \in \Re^n$ is the vector of output frequencies. The input $u = k(r - x) \in \Re^n$ is a function of reference frequencies and real frequencies of the nodes in the distributed system. It is important to note that relations between the frequencies of the n nodes lead to the system (1) is schedulable with respect to the use of processors, that is,

$$U = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

c_i are the consumes and p_i are de periods of task i.

Therefore it is possible to control the system through the input vector u such that the outputs y are in a region L non-linear where the system is schedulable. This is that during the time evolution of the

system (1) the output frequencies could be stabilized by a controller within the schedulability region L. This region could be unique or a set of subregions L_i in which each y_i converges, defined by:

$$\beta_1 \le y_1 \le \alpha_1$$
$$\beta_2 \le y_2 \le \alpha_2$$
$$\dots$$
$$\beta_2 \le y_2 \le \alpha_2$$

Each α_i and β_i belongs to minimum and maximum frequencies respectively to the node n_i which vary according to particular case study. Each node of the system starts with a frequency f_i and the LQR controller modifies the period $p_i = \frac{1}{f_i}$ of each task into a schedulable region L which is within α and β.regions. After control action the real frequency f_i of the node n_i is modified to $f_i^{'}$, it means that p_i in the time t_0 changes to $p_i^{'}$ at time t_1 to converge in a region where the system performance is close to optimal.

Time diagram of system dynamics (figure 1) shows the LQR controller modifications to set the task periods into region L.

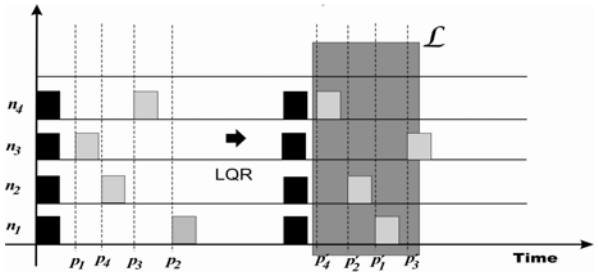

FIGURE 1. Task periods controlled by a LQR controller into a schedulable region.

The objective of controlling the frequency is to achieve coordination through the convergence of values.

2.1 Matrix coefficients proposal

Let $a_{ij} \in A$ given by a function of minimal frequencies f_m of node i and $b_{ij} \in B$ given by a function of maximal frequencies f_x:

$$a_{ij} = \varphi\left(f_m^1, f_m^2, \dots, f_m^n\right) = \varphi\left(f_m\right)$$
$$b_{ij} = \psi\left(f_x^1, f_x^2, \dots, f_x^n\right) = \psi\left(f_x\right)$$

The control input is given by a function of the minimal frequencies and the real frequencies of node i that is:

$$u = h(r - x) = k\left(f_m - f_r\right) \tag{2}$$

f_m, f_r, f_x are the vectors:

$$f_m = \left[f_m^1, f_m^2, \dots, f_m^n\right]^{\mathrm{T}}$$
$$f_r = \left[f_r^1, f_r^2, \dots, f_r^n\right]^{\mathrm{T}}$$
$$f_x = \left[f_x^1, f_x^2, \dots, f_x^n\right]^{\mathrm{T}} \tag{3}$$

Thus

$$\dot{x} = Ax + Bu$$
$$\dot{x} = Af_r + B\left(k\left(f_m - f_r\right)\right) \tag{4}$$

The elements of the matrices for system (1) are defined as follows:

$$a_{ij}=\begin{cases}\dfrac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^i} & i=j\\ \dfrac{f_m^j}{f_m^i} & i\neq j\end{cases}\qquad b_{ij}=\begin{cases}\dfrac{1}{f_x^i} & i=j\\ 0 & i\neq j\end{cases}\qquad c_{ij}=\begin{cases}1 & i=j\\ 0 & i\neq j\end{cases}$$

$\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)$ is the greatest common divisor of the minimum frequencies.

The, the system (1) can be written as:

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}\dfrac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^1} & \dfrac{f_m^2}{f_m^1} & \dfrac{f_m^3}{f_m^1} & \dfrac{f_m^4}{f_m^1}\\ \dfrac{f_m^1}{f_m^2} & \dfrac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^2} & \dfrac{f_m^3}{f_m^2} & \dfrac{f_m^4}{f_m^2}\\ \dfrac{f_m^1}{f_m^3} & \dfrac{f_m^2}{f_m^3} & \dfrac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^3} & \dfrac{f_m^4}{f_m^3}\\ \dfrac{f_m^1}{f_m^4} & \dfrac{f_m^2}{f_m^4} & \dfrac{f_m^3}{f_m^4} & \dfrac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^4}\end{bmatrix}\begin{bmatrix}f_r^1\\f_r^1\\f_r^1\\f_r^1\end{bmatrix}$$

$$+\begin{bmatrix}\dfrac{1}{f_x^1} & 0 & 0 & 0\\ 0 & \dfrac{f_r^2}{f_x^2} & 0 & 0\\ 0 & 0 & \dfrac{1}{f_x^3} & 0\\ 0 & 0 & 0 & \dfrac{1}{f_x^4}\end{bmatrix}\begin{bmatrix}k_1 & 0 & 0 & 0\\ 0 & k_2 & 0 & 0\\ 0 & 0 & k_3 & 0\\ 0 & 0 & 0 & k_4\end{bmatrix}\left(\begin{bmatrix}f_m^1\\f_m^2\\f_m^3\\f_m^4\end{bmatrix}-\begin{bmatrix}f_r^1\\f_r^2\\f_r^3\\f_r^4\end{bmatrix}\right)$$

$$\begin{bmatrix}y_1\\y_2\\y_3\\y_4\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}f_r^1\\f_r^2\\f_r^3\\f_r^4\end{bmatrix}$$

3. Case of study.

Case study is a 2-DOF helicopter system integrated to a CanBus network, detailed information can be found in [9]. The tasks for the NCS are set as follows, 1 ms as sampling period of sensor task and, controller and actuator tasks are event driven.

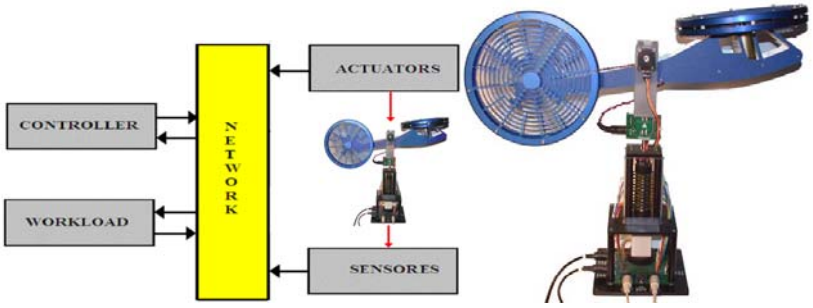

FIGURE 2. Networked Control System with workload

The Case study is a MIMO nonlinear, open-loop unstable and time varying system. The 2-DOF Helicopter experiment consists of a helicopter model mounted on a fixed base with two propellers that are driven by DC motors and is shown in Figure 2. The front propeller controls the elevation of the helicopter nose about the pitch axis and the back propeller controls the side to side motions of the helicopter about the yaw axis. The pitch and yaw angles are measured using high-resolution encoders. The pitch encoder and motor signals are transmitted via a slip ring. This eliminates the possibility of wires tangling on the yaw axis and allows the yaw angle to rotate freely about 360 degrees.

4. Numerical simulations

We performed numerical simulations of the system (1) without control and with LQR controller for values of maximum, minimum and real frequencies as following:

TABLE 1. Maximum, minimum, and real frequencies.

Node	Max freq.	Min. freq.	Real freq
1	60	40	55
2	50	30	50
3	50	10	25
4	45	25	30

The coefficient of matrix A and its eigenvalues are:

$$A=\begin{bmatrix} 0.1250 & 0.7500 & 0.2500 & 0.6250 \\ 1.3333 & 0.1667 & 0.3333 & 0.8333 \\ 4.0000 & 3.0000 & 0.5000 & 2.5000 \\ 1.6000 & 1.2000 & 0.4000 & 0.2000 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= 3.2536 \\ \lambda_2 &= -0.5885 \\ \lambda_3 &= -0.8585 \\ \lambda_4 &= -0.8150 \end{aligned}$$

4.1 LQR Control

We chose weight matrices $Q, R \in \Re^{4x4}$ as follows:

$$Q=\begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \qquad R=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the gain K $K \in \Re^{4x4}$ and $A_c = (A - BK) \in \Re^{4x4}$ are:

$$K=\begin{bmatrix} 137.32 & 103.95 & 4042 & 87.32 \\ 1124.75 & 94.62 & 34.41 & 79.38 \\ 45.42 & 34.41 & 12.70 & 28.90 \\ 130.98 & 99.23 & 36.12 & 83.5 \end{bmatrix} \qquad A_c=\begin{bmatrix} -2.3251 & -1.10360 & -0.4237 & -0.9347 \\ -1.3398 & -1.8586 & -0.4017 & -0.8683 \\ 3.0298 & 2.2650 & 0.2332 & 1.8824 \\ -1.1727 & -0.9008 & -0.3624 & -1.5650 \end{bmatrix}$$

and eigenvalues of A_c are

$$\begin{aligned} \lambda_1 &= -3.2536 \\ \lambda_2 &= -0.5997 \\ \lambda_3 &= -0.8606 \\ \lambda_4 &= -0.8182 \end{aligned}$$

Therefore the frequencies could be managed into region L. The implementation of this frequency transition model was included into helicopter model simulated on truetime. Four sensor sample pitch, yaw, pitch derivative and yaw derivative signals with period of 0.030 ms. The figure 3 shows the system instability for the angles pitch, yaw, and derivatives through 25 s.

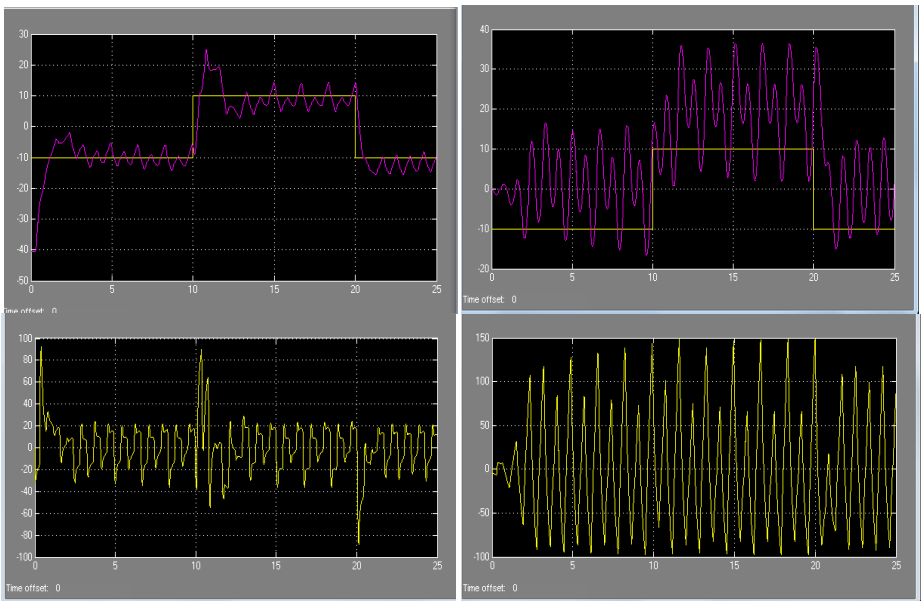

FIGURE 3. Response of 2-DOF Helicopter. Pitch and Yaw angles and derivatives

Using the system frequency transition model starting in the second 10 the system becomes stable due to change in the frequency data transmission of the sensors.

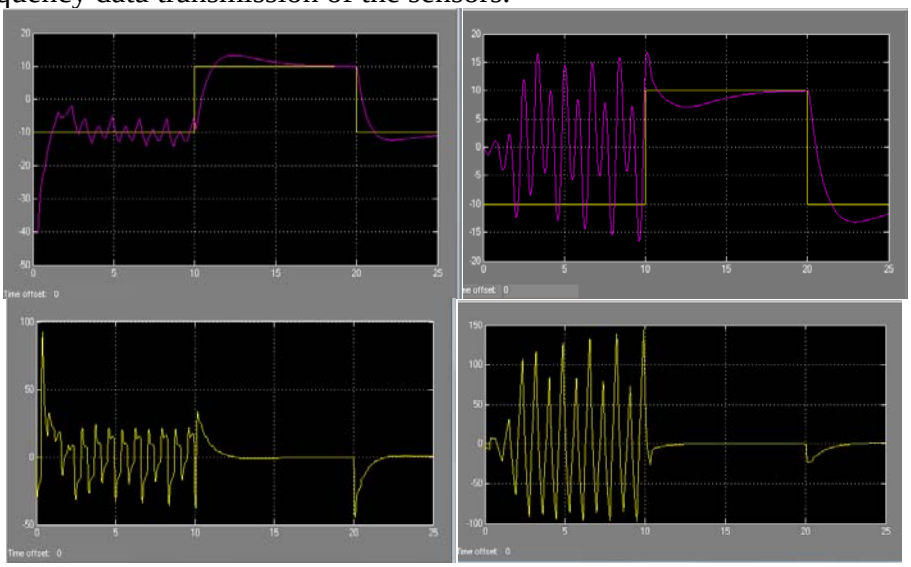

FIGURE 4. Response of 2-DOF Helicopter. Pitch and Yaw angles and derivatives using frequency transition model

The frequency controller modifies the frequencies into the limits defined by minimal and maximal frequencies.

5 Conclusions

This work shows a linear time invariant model of nodes frequency transmission involved into a distributed system. The significance of control the frequencies stem from the system schedulability. The key feature

of LQR control approach is a simple design with good robustness and performance capabilities lets to modify the frequencies easily. It has shown via numerical simulations the performance of the proposed control scheme through a NCS embedded on a helicopter prototype model.

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