Frequency Transition based upon dynamic Consensus for a Distributed System

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Abstract: This paper provides a strategy to schedule a real time distributed system. Modifications on frequency transmission (task periods) of system's individual components impact on system quality performance. In this work the authors propose a dynamic linear time invariant model based upon frequency transmission of agents in a distributed system and using LQR control approach to bring the system into a nonlinear region. Numerical simulations show the effectiveness of the LQR feedback control law to modify agent's frequency transmission.

Keywords: Distributed systems, Frequency transmission, Consensus, Control, Agents.

1 Introduction

At the present time distributed systems are widely used in the industrial and research. These systems fulfills critical mission and long-running applications, some characteristics of the distributed systems are either capacity to maintain consistency or recovering without suspending their execution. These systems should complete time restrictions, coherence, adaptability and stability among others. A current application on Distributed Systems under time restrictions are Networked Control Systems (NCS) which implementation consist of several nodes doing some part of the control process, sensor-actuator activities works under real time operating system and real time communication network. In order to achieve the objectives of all tasks performed, it is necessary for all agents of the system to exchange their own information through communication media properly. Therefore the mechanism of communication plays an important role on stability and control system performance implemented over A
a communication network [1].

1.1 Sampling and transmission rates

Network scheduling is a priority in the design of a NCS when a group of agents are linked through the available network resources. If there is not coordination among agents data transmissions may occur simultaneously and someone has to back off to avoid collisions or bandwidth violations. This results in transmission of some real time with delay or even failure to comply their deadlines. Therefore is necessary a scheduling control algorithm which minimize this loss of system performance [2]. Nevertheless there isn't a global scheduler that guarantees an optimal performance [3].

Mainly due to communication network introduce a several issues not deal properly, Lian *et al.* [4],[5] have designed some methodologies for networked nodes (agents of the system) to generate proper control actions and utilize communication bandwidth optimally. Frequency transmission could be obtained through the sampling rate. The task period p performed by an agent defines the frequency transmission, it means, $f = \frac{1}{p}$. Figure 1 shows that effectiveness of the networked control systems depends on the sampling rate, a region which control performance is acceptable deals with two points b and c associated to f_b and f_c sampling rates respectably which can be determined by characteristics an statistics of networked induced delays and device processing time delays. f_b implies that small sampling periods could be necessary to guarantee a certain level of control performance, as the sampling rate gets faster f_c, the network traffic load becomes heavier, the possibility of more contention time or data loss increase in a bandwidth-limited network and longer time delays result.

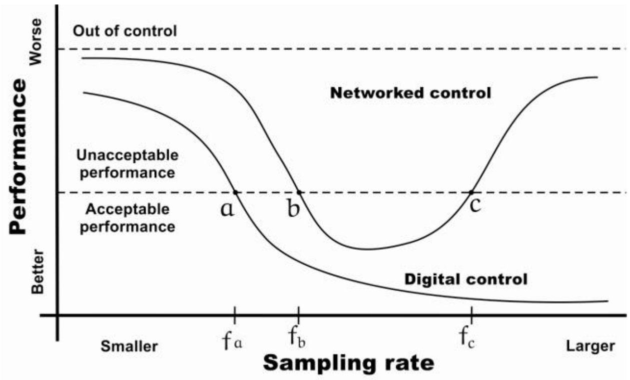

Fig. 1. Networked control system performance.

Hence it's very important to considerate either sampling periods or frequency transmission to obtain better system performance.

1.2 Consensus

Other hand, the basic idea of a consensus algorithm is to impose similar dynamics on the information states of each agent involved in a dynamical system [6]. In networks of agents consensus means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents. A consensus algorithm is an interaction rule that specifies the information exchange between an agent and all of its neighbors on the network. Recently several problems related to multi-agent networked systems with close ties to consensus problems have got an important interest. Olfati-Saber *et al.* in [7] present an overview of the key results on theory and applications of consensus problems in networked systems which include control theoretic methods for convergence and performance analysis of consensus protocols. Hayashi *et al.* [8] propose a fair quality service control with agent-based controller where each agent manages an allocated resource and a quality service level of several tasks working on a real time system, considers an application of typical consensus problem to fair quality service control in soft real time systems. The states of the system are resources allocated to a task such as CPU utilization, network bandwidth, and memory size, while the performance value of each agent is characterized by the performance function.

This paper shows a way to control the frequency of transmission among agents in a NCS based on their frequency transmission relations. We propose a lineal time invariant model in which the coefficients of the state matrix are the relations between the frequencies of each node and we use a LQR feedback controller that modifies transmission frequencies bounded between maximum and minimum values of transmission in which ensures the system's schedulability.

The rest of this paper is organized as follow, section 2 shows a frequency transmission model and a proposal to matrix coefficients of the model, section 3 presents a particular networked control system as a case of study, section 3 shows numerical simulations of the model presented and performance of LQR controller. Brief conclusions are presented at the end.

2 Frequency model

Let a distributed system with n nodes or agents that perform one task t_i with period p_i and consumption c_i each one for $i = 1,2,...,n$. The distributed system dynamics can be modeled as a linear time-invariant system, which state variables $x_1, x_2,..., x_n$ .are the frequencies of transmission $f_i = \frac{1}{p_i}$ from n nodes involved on it.

The authors assume there is a relationship between frequencies $f_1, f_2,..., f_n$ and external input frequencies $u_1, u_2,..., u_n$ which serve as coefficients of the linear system We assume there is a relationship between frequencies $f_1, f_2,..., f_n$ and external input frequencies $u_1, u_2,..., u_n$ which serve as coefficients of the linear system:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

(1)

$A \in \Re^{nxn}$ is the matrix of relationships between frequencies of the nodes, $B \in \Re^{nxn}$ is the scale frequencies matrix, $C \in \Re^{nxn}$ is the matrix with frequencies ordered, $x \in \Re^{n}$ is a real frequencies vector, $y \in \Re^{n}$ is the vector of output frequencies. The input $u = h(r - x) \in \Re^{n}$ is a function of reference frequencies and real frequencies of the nodes in the distributed system.

It is important to note that relations between the frequencies of the n nodes lead to the system (1) is schedulable with respect to the use of processors, that is,

$$U = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

Therefore it is possible to control the system through the input vector u such that the outputs y are in a region L non-linear where the system is schedulable. This is that during the time evolution of the system (1) the output frequencies could be stabilized by a controller within the schedulability region L. This region could be unique or a set of subregions L_i in which each y_i converges, defined by:

$$\beta_1 \le y_1 \le \alpha_1$$

$$\beta_2 \le y_2 \le \alpha_2$$

$$\dots$$

$$\beta_2 \le y_2 \le \alpha_2$$

However, it is not need $\alpha_n \le \alpha_{n-1} \le, \dots, \le \alpha_2 \le \alpha_1$ or $\beta_n \le \beta_{n-1} \le, \dots, \le \beta_2 \le \beta_1$.Each α_i and β_i belongs to minimum and maximum frequencies respectively for the node n_i which vary according to particular case study. Figure 2 shows the dynamics of the frequency system and the desired effect by controlling it through a LQR controller and defining a common region L for a set of frequencies. Each node of the system starts with a frequency f_i and the LQR controller modifies the period $p_i = \frac{1}{f_i}$ of each task into a schedulable region L .The real frequency f_i of the node n_i is modified to $f_i^{'}$, it means that p_i in the time t_0 changes to $p_i^{'}$ at time t_1 to converge in a region where the system performance is close to optimal.

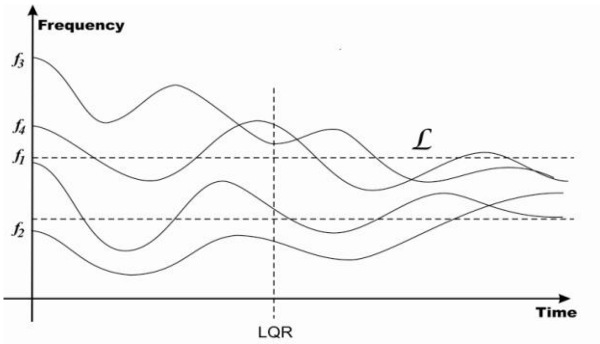

Fig. 2. Frequencies controlled by a LQR controller into a schedulability region.

Figure 3 shows a time diagram of system dynamics and the desirable effect of a LQR controller modifications to set the task periods into region L.

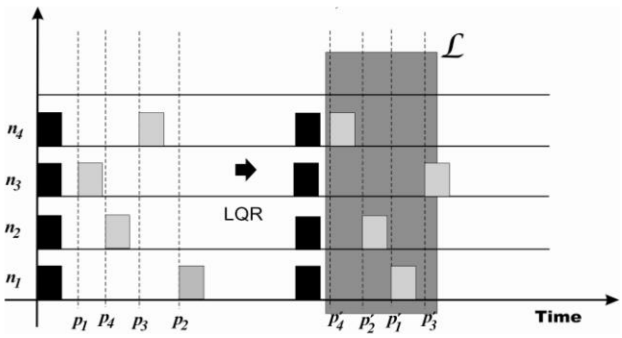

Fig. 3. Task periods controlled by a LQR controller into a schedulability region

The objective of controlling the frequency is to achieve coordination through the convergence of values.

2.1 Matrix coefficients proposal

Let $a_{ij} \in A$ given by a function of minimal frequencies f_m of node i and $b_{ij} \in B$ given by a function of maximal frequencies f_x:

$$a_{ij} = \varphi\left(f_m^1, f_m^2, ..., f_m^n\right) = \varphi\left(f_m\right)$$

$$b_{ij} = \psi\left(f_x^1, f_x^2, ..., f_x^n\right) = \psi\left(f_x\right)$$

The control input is given by a function of the minimal frequencies and the real frequencies of node i :

$$u = h(r - x) = k\left(f_m - f_r\right) \tag{2}$$

f_m and f_x are the vectors:

$$f_m = \left[f_m^1, f_m^2, \dots, f_m^n\right]^T$$
$$f_r = \left[f_r^1, f_r^2, \dots, f_r^n\right]^T$$

Then, the system (1) can be written as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ \dot{x} &= Af_r + B\left(k\left(f_m - f_r\right)\right) \\ \dot{x} &= Af_r + Bkf_m - Bkf_r \\ \dot{x} &= \left(A - Bkf_m\right) + Bkf_r \end{aligned} \quad (3)$$

3 Case of study

The authors consider a distributed system which performs a control close loop dynamic system based upon: sensor-controller-actuator and a centralized scheduler. Figure 4 shows the networked control system which consists of 8 processors with real-time kernel, connected by through a network type CSM / AMP (CAN) with a rate of sending data of 10000000 bits / s and not likely to data loss.

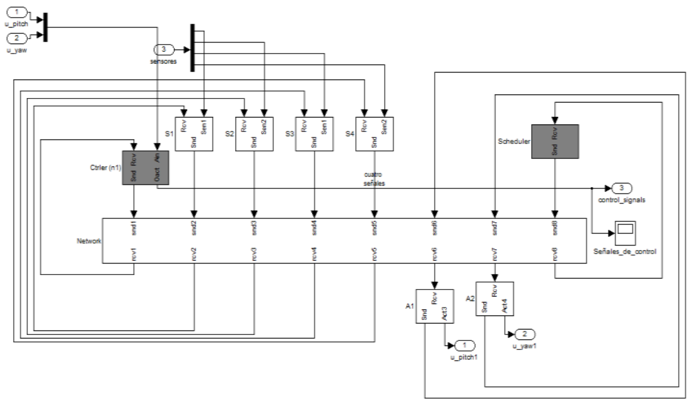

Fig. 4: Networked Control System.

These blocks of real-time kernel and network are simulated using Truetime [9],[10]. The first agent in the model, on the extreme left is the controller agent that uses the values from sensors and calculates control outputs. Sensor agents sample the analog signals. Two actuator agents located to the far right below, receives signals. Finally scheduler, main agent, above far right node, organizes the activity of other 7 agents and it is responsible for periodic allocation bandwidth used by others agents.

Focused on sensor agents which use a common communication media and performing closed loop control, each one has a real transmission frequency f_r and sets the minimum frequencies f_m and maximum f_x between which each node could transmit.

The regions $L_1 \bigcup L_2 \bigcup L_3 \bigcup L_4 \subseteq L$ as a whole should meet the following restriction

$$U = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

Where c_i is the time packet communication and it is value is 2 *ms* on average for Ethernet.

Elements of the matrices for system (1) are defined as follows:

$$a_{ij} = \begin{cases} \dfrac{\overline{\lambda}\left(f_m^1, f_m^2, f_m^3, f_m^4\right)}{f_m^i} & i = j \\ \\ \dfrac{f_m^j}{f_m^i} & i \neq j \end{cases} \qquad b_{ij} = \begin{cases} \dfrac{1}{f_x^i} & i = j \\ \\ 0 & i \neq j \end{cases} \qquad c_{ij} = \begin{cases} 1 & i = j \\ \\ 0 & i \neq j \end{cases}$$

$\overline{\lambda}\left(f_m^1, f_m^2, f_m^3, f_m^4\right)$ is the greatest common divisor of the minimum frequencies.

Due to:

$$f_m = \left[f_m^1, f_m^2, \ldots, f_m^n\right]^{\mathrm{T}}$$
$$f_r = \left[f_r^1, f_r^2, \ldots, f_r^n\right]^{\mathrm{T}}$$
$$f_x = \left[f_x^1, f_x^2, \ldots, f_x^n\right]^{\mathrm{T}}$$

also

$$x = f_r$$
$$u = k\left(f_m - f_r\right)$$

Using (3) we can write (1) as:

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}\frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^4\right)}{f_m^1} & \frac{f_m^2}{f_m^1} & \frac{f_m^3}{f_m^1} & \frac{f_m^4}{f_m^1}\\ \frac{f_m^1}{f_m^2} & \frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^4\right)}{f_m^2} & \frac{f_m^3}{f_m^2} & \frac{f_m^4}{f_m^2}\\ \frac{f_m^1}{f_m^3} & \frac{f_m^2}{f_m^3} & \frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^4\right)}{f_m^3} & \frac{f_m^4}{f_m^3}\\ \frac{f_m^1}{f_m^4} & \frac{f_m^2}{f_m^4} & \frac{f_m^3}{f_m^4} & \frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^4\right)}{f_m^4}\end{bmatrix}\begin{bmatrix}f_r^1\\f_r^2\\f_r^3\\f_r^4\end{bmatrix}$$

$$+\begin{bmatrix}\frac{1}{f_x^1} & 0 & 0 & 0\\ 0 & \frac{1}{f_x^2} & 0 & 0\\ 0 & 0 & \frac{1}{f_x^3} & 0\\ 0 & 0 & 0 & \frac{1}{f_x^4}\end{bmatrix}\begin{bmatrix}k_1 & 0 & 0 & 0\\ 0 & k_2 & 0 & 0\\ 0 & 0 & k_3 & 0\\ 0 & 0 & 0 & k_4\end{bmatrix}\left(\begin{bmatrix}f_m^1\\f_m^2\\f_m^3\\f_m^4\end{bmatrix}-\begin{bmatrix}f_r^1\\f_r^2\\f_r^3\\f_r^4\end{bmatrix}\right)$$

$$\begin{bmatrix}y_1\\y_2\\y_3\\y_4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}f_r^1\\f_r^2\\f_r^3\\f_r^4\end{bmatrix}$$

that is:

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}\frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^1} & \frac{f_m^2}{f_m^1} & \frac{f_m^3}{f_m^1} & \frac{f_m^4}{f_m^1}\\ \frac{f_m^1}{f_m^2} & \frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^2} & \frac{f_m^3}{f_m^2} & \frac{f_m^4}{f_m^2}\\ \frac{f_m^1}{f_m^3} & \frac{f_m^2}{f_m^3} & \frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^3} & \frac{f_m^4}{f_m^3}\\ \frac{f_m^1}{f_m^4} & \frac{f_m^2}{f_m^4} & \frac{f_m^3}{f_m^4} & \frac{\bar{\lambda}\left(f_m^1,f_m^2,f_m^3,f_m^3\right)}{f_m^4}\end{bmatrix}\begin{bmatrix}f_r^1\\f_r^1\\f_r^1\\f_r^1\end{bmatrix}$$

$$+\begin{bmatrix}k_1\,\frac{f_m^1-f_r^1}{f_x^1}\\ k_2\,\frac{f_m^2-f_r^2}{f_x^2}\\ k_3\,\frac{f_m^3-f_r^3}{f_x^3}\\ k_4\,\frac{f_m^4-f_r^4}{f_x^4}\end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_r^1 \\ f_r^2 \\ f_r^3 \\ f_r^4 \end{bmatrix}$$

4 Numerical simulation

We performed numerical simulations of the system (1) without control and with LQR controller for values of maximum, minimum and real frequencies as following:

Table 1. Maximum, minimum, and real frequencies.

Node	Max freq.	Min. freq.	Real freq
1	65	40	53
2	55	35	52
3	55	15	33
4	45	30	38

thus

$$A = \begin{bmatrix} 0.1250 & 0.7500 & 0.2500 & 0.6250 \\ 1.3333 & 0.1667 & 0.3333 & 0.8333 \\ 4.0000 & 3.0000 & 0.5000 & 2.5000 \\ 1.6000 & 1.2000 & 0.4000 & 0.2000 \end{bmatrix}$$

and eigenvalues

$$\lambda_1 = 3.1937$$
$$\lambda_2 = -0.7149$$
$$\lambda_3 = -0.8676$$
$$\lambda_4 = -0.8435$$

The system is unstable

4.1 LQR Control

We chose weight matrices $Q, R \in \Re^{4x4}$ as follows:

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the gain K $K \in \Re^{4x4}$ and $A_c = (A - BK) \in \Re^{4x4}$ are:

$$K = \begin{bmatrix} 118.90 & 104.41 & 47.01 & 90.00 \\ 123.39 & 108.54 & 48.82 & 93.47 \\ 55.56 & 48.82 & 22.11 & 42.08 \\ 130.01 & 114.25 & 51.44 & 98.62 \end{bmatrix} \qquad A_c = \begin{bmatrix} -1.70 & -0.73 & -0.34 & -0.63 \\ -1.10 & -1.83 & -0.45 & -0.84 \\ 1.65 & 1.44 & -0.06 & 1.23 \\ -1.55 & -1.37 & -0.64 & -2.02 \end{bmatrix}$$

and eigenvalues of A_c

$$\lambda_1 = -3.1937$$
$$\lambda_2 = -0.7199$$
$$\lambda_3 = -0.8690$$
$$\lambda_4 = -0.8460$$

Figure 5 shows the dynamics of the controlled system

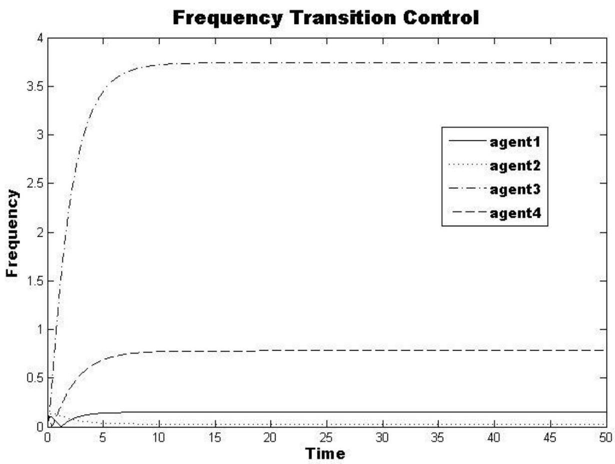

Fig. 5. Frequency response controlled by a LQR controller.

The LQR controller could modify frequencies into the limits defined by minimal and maximal frequencies.

5 Conclusions

In this work, we have present a linear time invariant model of nodes frequency transmission involved into a distributed system. The significance of control the frequencies stem from the system schedulability. The key feature of LQR control approach is a simple design with good robustness and performance capabilities, easily frequencies are modified. We have shown via numerical simulations the performance of the proposed control scheme.

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