Real-Time Distributed Control
A Diverse Approach for a Non-Linear Problem

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Abstract:
Nowadays the study of faults and their consequences becomes an issue into highly safety critical computer
network systems. How to bound the effects of a fault and how to tackle them into a dynamic system is still an open
field. In here, an approach to study this problem is presented as hybrid strategy. The use of a multi-model
technique from the point of view of real-time systems and Intelligent Control structure are given in order to
accomplish one challenge, to overcome the presence of local faults and the respective time delays within a real-
time distributed system. This approach is pursued as reconfigurable strategy according to communication time
delays within a real-time distributed system. In fact, it is pursued as reconfigurable strategy according to
communication delays and local faults where control strategy is modified from several perspectives.

1. Introduction
Control reconfiguration is presented as an available approach for fault coverage in order to keep system
performance. In here reconfiguration is pursued as response of time delay modification rather than fault
appearance although this is the basis for control reconfiguration.
Several strategies for managing time delay within control laws have been studied by different research
groups. For instance, Nilsson (1998) proposes the use of a time delay scheme integrated to a reconfigurable
control strategy based upon a stochastic methodology. Another strategy has been proposed by Jiang et al.,
(1999) where time delays are used as uncertainties, which modify pole placement of a robust control law.
Izadi et al., (1999) present an interesting case of fault tolerant control approach related to time delay
coupling. Blanke et al., (2003) have studied reconfigurable control from the point of view of structural
modification. They stablish a logical relation between dynamic variables and the respective faults. On the
other hand Benítez-Pérez et al., (2005) and Thompson (2004) consider that reconfigurable control performs a
combined modification of system structure and dynamic response. It has present the advantage of bounded
modifications over system response.
Present approach takes time delays due to communication as deterministic measured variables as well as
actuator fault presence by modification of system parameters affecting local control with two conditions,
loosing local peripheral elements and the related time delays. In here, control law views time delays as a
result of deterministic reconfigurable communications based upon scheduling algorithm.
Some considerations need to be stated in order to define this approach. Firstly, faults are strictly local in
peripheral elements and these are tackled by just eliminating the faulty element. In fact, faults are
catastrophic and local. Time delays are bounded and restrictive to scheduling algorithms. Global stability can
be reached by using classical control strategy.
The objective of this paper is to present a strategy for control reconfiguration based upon time delay
knowledge as well as local fault effects within a distributed system environment considering the magnetic
levitation challenge. The novelty of this work is to propose different strategies for reconfiguration based on
fault appearance and consequent time delays. Specifically three strategies are reviewed Model Predictive
Control (MPC), Fuzzy Logic Control (FLC) and Fuzzy Model Predictive Control (FMPC) (Abonyi, 2003).
Since reconfiguration is a consequence of a complex problem two kind of modeling strategies are followed.
First computer network is designed by the use of Real Time Systems (RTS), second, control design is taken
into account by a strategy of intelligent control.

2. Structural Reconfiguration Algorithm
This paper is placed as a strategy for reconfigurable systems as shown in Fig. 1. It is focused into
reconfigurable control law due to the presence of local faults and time delays as consequences. The effects of
local faults are out the scope of this paper. Time delays are measurable and bounded according a real-time
scheduling algorithm (Liu, 2000). Several algorithms can be pursued such as Rate Monotonic, Deadline
Monotonic or Earliest Deadline First (EDF) (Lian et al., (2002), Benítez-Pérez et al., (2003) and Liu et al.,
(1973)). The use of last algorithm is pursued in here due to flexibility of task reorganization during online
performance. Basic procedure of EDF requires several characteristics from each task such as deadlines, consumption times and priorities.

Fig. 1 represents the different stages required to perform reconfiguration. First stage takes place when fault is detected (step 1), thereafter, structural reconfiguration is required (step 2) and plan validation is performed (step 3). If the selected plan is valid, it is sent to the rest of the computer nodes in order to perform the computer network reconfiguration (step 4). At the same time, the related control law is chosen (step 4’) as part of second stage. Furthermore, this control law takes place as well as new configuration plan (steps 5 and 6).

The communication network plays a key role in order to define the behaviour of the dynamic system in terms of time variance from communications and processing although it presents a nonlinear behaviour. In order to understand such a nonlinear behaviour, time delays are incorporated by the use of real-time system theory that allows time delays to be bounded even in the case of causal modifications due to external effects. As mention in last section three modeling techniques are implemented in order to determine a holistic view of the reconfiguration effect. The use of these two models (Dynamic Scheduling and Intelligent Control) for holistic reconfiguration allows different vistas of the same needs. In order to accomplish reconfiguration, it is proposed to determine how time delays modify control structure and how these can be bounded. In here the followed method is the control law definition, automaton design and scheduling restrictions.

The core of these modeling approaches is a way to bound time delays. In particular by using a scheduling algorithm. The objective of this is to perform on-line control reconfiguration based upon a review of the proposed plan following EDF approximation. The review uses a ART2 network (Frank et al., 1998) in order to classify valid and non-valid plans. First, the ART2 network is trained offline using valid and non-valid plans from EDF evaluation and case study response. Based on this training procedure two main regions are determined, one related to suitable reconfigurations and other that holds non-trustable reconfigurations. During online stage ART2 network allows classification from new plans if the response of this network is within the region of valid plans the computer network will be reconfigured otherwise the proposed plan will be rejected (Fig. 2).
During offline procedure ART2 network is trained by selection of valid plans according to EDF. Second stage is an online procedure to test system response where a reconfiguration plan is proposed and evaluated. The study and evaluation of ART2 neural network as decision maker is out the scope of this paper, the interested reader can refer to García-Zavala et al., (2005) and Quiñones-Reyes et al., (2006).

To define the communication network performance, the use of the true-time network is pursued. This strategy achieves network simulation based upon message transactions that are based on the real-time toolbox from MATLAB. Extended information from this tool is available at (True-Time 2003); the true time main characteristics are shown next. In the true time model, computer and network blocks are introduced in Figure 3.

These blocks are event driven, and the scheduling algorithm is managed by the user independently of each computer block. True time simulation blocks are basically two blocks. These have been developed by the Department of Automatic Control, Lund Institute of Technology, Sweden. Each kernel represents the interface between the actual dynamical model and the network simulation. Here, continuous simulation and digital conversion take place to transmit information through the network. This tool provides the necessary interruptions to simulate delay propagation as well as synchronization within the network.

There is one consideration called task consumption time that can be performed with no apparent restriction; nevertheless, this is fixed by communication performance and its respective priorities. As an example of multirate system, Table 1 is proposed, taking into account the related time graph in Figure 4. Where schemantine procedure is shown in Fig. 5 (Benitez-Perez et al., 2005).

<table>
<thead>
<tr>
<th>Elements</th>
<th>Consumption Time</th>
<th>Period (ms)</th>
<th>Communication Time Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>s1</td>
<td>c_{s1}</td>
<td>T_{1}</td>
<td>τ_{s1c}</td>
</tr>
<tr>
<td>s2</td>
<td>c_{s2}</td>
<td>T_{2}</td>
<td>τ_{s2c}</td>
</tr>
<tr>
<td>s3</td>
<td>c_{s3}</td>
<td>T_{3}</td>
<td>τ_{s3c}</td>
</tr>
<tr>
<td>c1</td>
<td>c_{c1}</td>
<td>T_{c1}</td>
<td>τ_{c1d1},τ_{c1d2},τ_{c1s1}</td>
</tr>
</tbody>
</table>
If a local failure occurs and the fault accommodation module is implemented outside, the affected fault stage modifies the communication performance as shown in Figure 6 and Table 2. In this case, new nodes appear and an extra sporadic time delay is added to the control system. Therefore, two strategies need to be taken into account such as the modification of the scheduling algorithm and the related control law modeling.

Table 2. Data used for the multivariable example considering the FM element

<table>
<thead>
<tr>
<th>Elements</th>
<th>Consumption Time</th>
<th>Period</th>
<th>Communication Time Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>s1</td>
<td>c_{s1}</td>
<td>T_1</td>
<td>τ_{s1c}</td>
</tr>
<tr>
<td>s2</td>
<td>c_{s2}</td>
<td>T_2</td>
<td>τ_{s2c}</td>
</tr>
<tr>
<td>s3</td>
<td>c_{s3}</td>
<td>T_3</td>
<td>τ_{s3c}</td>
</tr>
<tr>
<td>FM</td>
<td>c_{FM}</td>
<td></td>
<td>τ_{FM}</td>
</tr>
<tr>
<td>c_1</td>
<td>c_{c1}</td>
<td>T_{c1}</td>
<td>τ_{c1c}, τ_{c1d1}, τ_{c1d2}</td>
</tr>
<tr>
<td>A_1 s_1</td>
<td>c_{A1} + c_{s1}</td>
<td>T_{A1}</td>
<td>τ_{s1e}</td>
</tr>
<tr>
<td>A_2</td>
<td>c_{A2}</td>
<td>T_{A2}</td>
<td></td>
</tr>
<tr>
<td>C_2</td>
<td>c_{c2}</td>
<td>T_{c2}</td>
<td>τ_{c2e}</td>
</tr>
<tr>
<td>A_3</td>
<td>c_{A3}</td>
<td>T_{A3}</td>
<td></td>
</tr>
</tbody>
</table>
Where communication time delays between elements are $\tau_{sc}$ (sensor to controller) and $\tau_{ca}$ (controller to actuator).

A common strategy to incorporate time delays is Model Predictive Control (MPC) (Yi et al., 2002) (Dardinier-Maron et al., 1999) (Mathworks, 1998) (Cervin et al., 1998); the objective of this strategy is to compute the future control sequence following the optimal $I$-step ahead prediction. $y(t+i|t)$ are driven close to $y(t+i)$ for a predicted horizon. The way to approach this condition is to follow an objective function $J$ that depends on present and future control signals and uncertainties. This strategy defines the observed system as the next linear representation

\[
\begin{pmatrix}
\mathbf{I} + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_m z^{-\infty} \\
+ b_0 z^{-1} + b_1 z^{-2} + \ldots + b_n z^{-\infty}
\end{pmatrix} y(t) = z^{-1} \begin{pmatrix}
b_0 z^{-1} + b_1 z^{-2} + \ldots + b_n z^{-\infty} \end{pmatrix} y(t-1)
\]

where $[a_1 \ldots a_m]$ are the components related to system output and $[b_0 \ldots b_n]$ are the components related to system input. Finally, $[c_1 \ldots c_m]$ are the components related to system error. The proposed objective function is

\[
J(N_2, N_0) = \sum_{j=N_2}^{N_1} \delta(j) (y_j(t+j) - w(t+j))^2 + \sum_{j=N_2}^{N_1} \lambda(j) (\Delta u(t+j - 1))^2
\]

where $N_2$ and $N_0$ are the minimum and maximum costing horizons, $\delta(j)$ and $\lambda(j)$ are weighting sequences, and $w(t+j)$ is the future trajectory to be followed. From this objective function, it is necessary to optimize the future control response $u(t), u(t+1), \ldots$, and so on, where the plant response $y(t+j)$ is closed to $w(t+j)$.

3. First Scheme

First modeling approach is based on control law modification taken into account time delay appearance, in particular for three band case study. The schematic setup is based on Fig. 7 considering system response and control implementation.
In Fig. 7, X is the linear displacement
\( \theta^* \) is the angular displacement
\( \Omega^* \) is the angular velocity
F is the lineal force
J* is the lineal inercy
k is the parameter
\( \tau^* \) is the torque
a is the radius
v is the lineal velocity

In this case, the plant presents two cases with or without a box per belt. As the second case is trivial, the first case is expressed per belt considering the mass of the box (referred to as \( m \)). The first conveyor belt is expressed as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{\theta}_1
\end{bmatrix} =
\begin{bmatrix}
J/m & 0 \\
0 & J_x
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
x_1
\end{bmatrix} - \begin{bmatrix}
1/m \\
0
\end{bmatrix} \tau_1
\]

(3)

The second conveyor and the third conveyor belt follow similar dynamics. From these considerations, discrete plants are defined next by considering the presence of the box

\[
x(k + l) = Ax(k) + \sum_{i=0}^{l} B_i u(k - i)
\]

(4)

where \( l=1 \) because the maximum number of sensors with delays is just one. Therefore, the \( A \) matrix is expressed as

\[
A = \begin{bmatrix}
\exp(J/m) & 0 \\
0 & \exp(J_x)
\end{bmatrix}
\]

(5)

where \( T \) is the inherent sampling period, and \( t_0^i \), \( t_1^i \), and \( t_2^i \) are the related delays of the plant. For the case of local control laws, these are expressed next as

\[
x_c(k + 1) = Ax_c(k) + B_c u_c(k)
\]

(6)

giving the delays as a result of decomposition from sensor and actuators, which are expressed as \( \tau_{sc} \) and \( \tau_{ca} \), respectively. The augmenting representation is given next:

\[
u_c(k) = y_c(k - \tau_{sc})
\]

\[
u_p(k) = y_p(k - \tau_{ca})
\]

(7)
where states are augmented as
\[ z = \begin{bmatrix} x_p(k) \\ x_c(k) \end{bmatrix} \]  
and expressed as
\[ z(k+1) = \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix} z(k) + \begin{bmatrix} 0 & B_c C_p \\ B_p D_c C_p & 0 \end{bmatrix} z(k - \tau_{sc}) + \begin{bmatrix} 0 & B_p C_c \\ 0 & 0 \end{bmatrix} z(k - \tau_{ca} - \tau_c) \]  
As it is possible to restructure eqn. 9 to define the stability of network control, the next configuration is as follows:
\[ F_i^j = \begin{bmatrix} A_i^j & 0 \\ 0 & A_i^j \end{bmatrix} \]
\[ F_i^j = \begin{bmatrix} 0 & 0 \\ B_i^j C_p & 0 \end{bmatrix} \quad F_2^j = \begin{bmatrix} B_i^j D_i^j C_p & 0 \\ 0 & 0 \end{bmatrix} \]
\[ F_3^j = \begin{bmatrix} 0 & B_i^j C_c \\ 0 & 0 \end{bmatrix} \]
Therefore, the state vector is modified for these time delays where the system is asymptotically stable based on \( F_i^j + \sum_{j=1}^{3} F_i^j \). Then, because a single control loop is stable, based on eqn 8, it is possible to define stability for every loop
\[ \tau < \frac{\sigma}{\delta \sum_{j=1}^{3} |F_i^j| \left( F_i^j + \sum_{j=1}^{3} F_i^j \right)} \] 
where \( \tau \) is the maximum value from all possible time delays at all loops.
\[ \sigma = \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \]
\[ \delta = \left[ \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \right]^{1/2} \]
where \( \lambda_{\max}(P) \) and \( \lambda_{\min}(Q) \) are the maximum and minimum eigenvalues of \( P \) and \( Q \) matrices, respectively. The proposed Lyapunov Equation
\[ \left( F_i^j + \sum_{j=1}^{3} F_i^j \right)^T P + P \left( F_i^j + \sum_{j=1}^{3} F_i^j \right) = -Q \] 
where \( P, Q \) are positive definite symmetric matrices and \( \lambda_{\min}, \lambda_{\max} \) are eigenvalues of the matrix, where
From control law expression, the related time delays are defined as $\tau_{sc}$, $\tau_{ca}$, $\tau_{c}$.

Having shown the local control laws structures, another modeling approach is the global structure in terms of an automaton (Fig. 8). Where reconfiguration is expressed for the formal event manager. In this case, two states are possible with several events, which are managed by the sensor vector for each belt (first, second, and third belts) and expressed as $S_{i}^{0}$, $S_{i}^{1}$, and $S_{i}^{2}$, respectively considering fault free scenario.

The switching effect is neglected in this fault-free scenario.

For the case of a fault scenario, a new state appears for global control (Fig. 9) related to the action pursued when a fault is presented. The necessary event for reaching such a state is $S_{i}^{3}$, and the fault’s last event is composed of local information given by each local sensor with a relation to the health condition measures.

\[
V(x) = \frac{1}{2} x^T(t) P x(t)
\]

\[
V(x) = \frac{1}{2} z^T(t) P z(t) + \frac{1}{2} z^T(t) P z(t) \leq -\frac{1}{2} z^T(t) Q z(t) + \left| z^T(t) P \sum_{i=1}^{m} F_i \int_{-\tau_i}^{0} F_i z^T(t+\theta) + \sum_{i=1}^{m} F_i z^T(t-\tau_i^+ + \theta) d\theta \right|
\]
In this case, time delays are defined in terms of transitions between states.

4. Second Scheme

Another way to observe this problem is considering individual modeling by the conjunction of several linear models and gain scheduling among them. In that respect, the use of TKS (Takagi-Sugeno Fuzzy System) MP (Model Predictive) Control approach is pursued in here. It is assumed that the fault is detectable and measurable (This is a condition in this paper) (Quiñones-Reyes et al., 2006).

Case study is a magnetic system integrated to a computer network as shown in Fig. 10 (Wincon, 2003).

![Magnetic Levitator Case Study](image)

Fig. 10. Magnetic Levitator Case Study

The dynamics of case study expressed in transfer function is:

\[
G_{hi}(s) = \frac{-k_{bd}w_b^2}{s^2 - w_b^2}
\]

\[
k_{bd} = \frac{x_{bo}}{I_{co}}
\]

\[
w_b = \sqrt{\frac{g}{x_{bo}}}
\]

where

- \( g \) is the gravity force
- \( I_{co} \) is the current of the coil
- \( x_{bo} \) is the distance from coil to ball position.

Following time delays considerations, they are implemented as follows:

\[
e^{-T_s} = \frac{2 - T_s}{2 + T_s}
\]

where \( T \) is the transport delay, that for fault-free scenario is defined as:

\[
T = t_s + 4 + t_{cm} + t_c + t_{cm} + t_a
\]

where:

- \( t_s \) is the time consumed by sensors
$t_{sc}^{cc}$ is the communication time between sensor and control
$t_s$ is the consumed time by control node
$t_{cm}$ is the communication time between controller and actuator
$t_a$ is the time consumed by actuator

For fault scenario, $T$ time is expressed as:

$$T = t_s + t_{sc}^{cc} + t_{cm} + t_t + t_{cm} + t_a$$  \hspace{1cm} (17)

where:

$t_s$ is the time consumed by sensors
$t_{sc}^{cc}$ is the communication time between sensor and control
$t_{cm}$ is the communication time between controller and actuator
$t_{cm}^{fat}$ is the communication time between sensor and fault tolerance module.
$t_t$ is the consumption time from fault tolerance module
$t_{cm}^{fc}$ is the time consumed for the fault sensor to send messages to its neighbor and produce agreement
$t_c$ is the time consumed by control node
$t_{cm}$ is the communication time between controller and actuator
$t_a$ is the time consumed by actuator

From both scenarios there is an element known as fault tolerance element that presents extra communication for control performance although it masks any local fault from sensors. As the time delays have been bounded, the plant model is defined based on eqns 18 and 19 and Fig. 11.

![Fig. 11. Plant and control law integration](image)

The proposed dynamic plant is based upon the following structure:

$$x(k+1) = a^p x(k) + B^p u(k)$$  \hspace{1cm} (18)

where $a^p \in \mathbb{R}^{n \times n}$, $c^p \in \mathbb{R}^{n \times 1}$ and $B^p \in \mathbb{R}^{n \times l}$ are matrices related to the plant. $x(k)$, $u(k)$ and $y(k)$ are the states, inputs and outputs respectively. Specially $B^p$ is stated as

$$B^p = \sum_{i=1}^{N} \rho_i B_i \sum_{j=1}^{M} \int_{t_j}^{t_{j-1}} e^{-a^p (t-\tau)} \, d\tau$$  \hspace{1cm} (19)
where $\rho_i = 1$ and $\sum_{i=1}^{N} \rho_i = 1$

taking into account that $N$ are the total number of possible faults and $M$ are the involved time delays from each fault. Current communication time delays are expressed as $\tau_{i-j}$ and $\tau_j$ remember that $\sum_{j=1}^{M} \tau_j \leq T$

following eqns 3 and 4, and $B_i$ is integrated as

$$B_i = \begin{bmatrix} b_{i1} \\ b_{i2} \\ \vdots \\ 0i \end{bmatrix}$$

where $b_{i1} \rightarrow b_{iN}$ are the elements conformed at the input of the plant (such as actuators) and $0_i$ is the lost element due to local actuator fault where $B^p_i$ represents only one scenario following eqn. 20. Current $B^p_i$ considers local actuator faults and related time delays of

$$B^p_i = B_i \sum_{j=1}^{M} \int \sum_{i} e^{-a_p (t-\tau)} d\tau$$

(20)

For simplicity purposes $B^p_i$ is used in order to depict local linear plants From this representation fuzzy plant is defined as follows taking into account each time delay and fault cases:

$$r_i \text{ if } x_1 \text{ is } \mu_{i1} \text{ and } x_2 \text{ is } \mu_{i2} \text{ and...and } x_i \text{ is } \mu_{in} \text{ then } a_i x(k) + B^p_i u(k)$$

and

$$h_i = \prod_{j=1}^{l} \mu_{ij}(x_j)$$

(22)

where $\{x_1, ..., x_i\}$ are current state measures, $l$ is the number of states, $i = \{1, ..., N\}$ is one of the fuzzy rules, $N$ is the number of the rules $r_i$ which is equal to the number of possible faults and $\mu_{ij}$ are the related membership functions which are gaussians defined as:

$$\mu_{ij}(x_j) = \exp \left(-\frac{(x_j - c_{ij})^2}{\sigma_{ij}^2} \right)$$

(23)

where $c_{ij}$ and $\sigma_{ij}$ are constants to be tuned. Final representation of plant as integrated system is based upon center of area defuzzification method.
It is important to remember eqn. 20 in order to pursue system response. From this representation of a global nonlinear system it is necessary to define global stability as a result of this fuzzy system. This review is given considering fuzzy logic control approach. The result of this system representation allows the integration of nonlinear stages and transitions to basically a group of linear plants. As from the point of view of the control approach, this is defined as Fuzzy Model Predictive Control as follows, taking the input of the plant as consequent:

\[
u^i = \left( S^T \mathbf{Q} S + \mathbf{R} \right)^{-1} S \mathbf{Q}(w - p)
\]

where \(w\) are the future set points, \(u^i\) is the control output, the matrices \(Q\) and \(R\) are positive definite weight matrices:

\[
\begin{align*}
\mathbf{Q} &= \text{Diag}(Q) \\
\mathbf{R} &= \text{Diag}(R)
\end{align*}
\]

\(S\) represents the effect of future outputs and from the integration to antecedent representation of Fuzzy System (eqn. 21)

\[
D_i = \prod_{j=1}^{N_p} \mu_j(y_j, u_j)
\]

and where \(N_p\) is the number of possible inputs for Fuzzy plant, \(y_i\) is the output of the plant. For the antecedent part of fuzzy control \(\Omega\)

\[
\Omega_i = \prod_{j=1}^{N_A} \mu_j(y_j, u_j)
\]

Where \(N_A\) is the number of possible inputs for fuzzy controller. The reader should remember that eqns 16 and 17 are the basis for time delay based upon the use of EDF scheduling algorithm presented as:

\[
\tau_{x(i)} + \tau_x + \tau_{\text{in}} + \tau_d < T
\]

From this perspective plant representation is given by:

\[
x(k+1) = \frac{\sum_{i=1}^{N} D_i(y, u) - \left( a^i x(k) + B^p_i u(k) \right)}{\sum_{i=1}^{N} D_i(y, u)}
\]

Where \(N\) is the number of rules, \(x(k+1)\) is the estimation of the states, and the input of the plant is defined as:

\[
u = \frac{\sum_{i=1}^{N} \Omega_i(y, u) - \left( S^T \mathbf{Q} S + \mathbf{R} \right)^{-1} S \mathbf{Q}(w - p_i)}{\sum_{i=1}^{N} \Omega_i(y, u)}
\]

Where the result is:
\[
x(k+1) = -\left( \sum_{i=1}^{N} D_i(y,u) \right) x(k) + B_i^p \left( \sum_{i=1}^{N} \left( (\Omega_i(y,u) - \left( a x(k) + B_i^p u(k) \right) \right) \right) + \left( \sum_{i=1}^{N} \Omega_i(y,u) \right) + \left( \sum_{i=1}^{N} \delta_i(u_i) \right)
\]

(30)

Where \( B_i^p \) has been defined in eqn 20.

Now from the point of view of MPC the cost function is defined as

\[
J = \sum_{i=1}^{N} B_i^p \left( r_i - y_i \right)^2 + \sum_{i=1}^{N} \left( \delta_i(u_i) \right)^2
\]

(31)

where \( r_i \) and \( y_i \) are the reference and output values respectively. This eqn can be seen as:

\[
J = \sum_{i=1}^{N} B_i^p \left( r_i - Cx(i-1) \right)^2 + \sum_{i=1}^{N} \left( \delta_i(u_i) \right)^2
\]

(32)

now considering the variable \( u_i \) defined as in eqn 25:

\[
J = \sum_{i=1}^{N} B_i^p \left( r_i - Cx(i-1) \right)^2 + \sum_{i=1}^{N} \left( \delta_i(u_i) \right)^2
\]

(33)

Remember that the values of \( Q, S \) and \( R \) positive definite matrices. Obtaining the partial derivatives for each variable in order to obtain the optimal values, where the optimization procedure of \( \delta, Q, R, c_i, \) and \( \sigma_i \) are left to the use since it can be performed by an exhaustive multivariable optimization technique like MOGA (Multi-Objective Genetic Algorithms).

5. Third Approximation

Another form to represent this complex situation of reconfiguration due to faults and time delays is through the use of pure TKS modelling as pursued next. Based on eqns. 18-24, fuzzy system is stated where state vector is calculated as

\[
r_i \text{ if } x_1 \text{ is } A_{i1} \text{ and } x_2 \text{ is } A_{i2} \text{ and...and } x_1 \text{ is } A_{i1} \text{ then } a_i^p x(k) + B_i^p u(k)
\]

(34)

For simplicity purposes \( B_i^p \) is used in order to depict local linear plants and \( A_{i} \) are the related membership functions. From this representation fuzzy plant is defined and the antecedent part is:

\[
h_i = \prod_{j=1}^{1} A_{i_j}(x_j)
\]

(35)

where \( \{x_1, ..., x_l\} \) are current state measures, \( l \) is the number of states, \( i = \{1, ..., N\} \) is one of the fuzzy rules \( N \) is the number of the rules which is equal to the number of possible faults and \( A_{i_j} \) are the related membership functions which are gaussians defined as:

\[
A_{i_j}(x) = \exp \left( -\left( \frac{x - c_{i_j}}{\sigma_{i_j}^2} \right)^2 \right)
\]

(36)

where \( c_{i_j} \) and \( \sigma_{i_j} \) are constants to be tuned. Final representation of plant as integrated system is based upon center of area defuzzification method as shown in equation 37.
\[ x^p(k+1) = \frac{\sum_{i=1}^{N} h_i \left( a_i^p x(k) + B_i^p u(k) \right)}{\sum_{i=1}^{N} h_i} \] (37)

It is important to remember eqn 19 in order to pursue system response. From this representation of a global nonlinear system as the integration of several linear systems it is necessary to define global stability as a result of this fuzzy system. This review is given in following section considering fuzzy logic control approach. The result of this system representation allows the integration of nonlinear stages and transitions to basically a group of linear plants.

From the representation of the plant as fuzzy system (Yi et al., 2002) it is necessary to develop the control law as a group of bounded local linear control laws related to each local linear system. The structure of each fuzzy rule is:

\[ r_i \quad \text{if} \quad x_1 \quad \text{is} \quad A_{i1}^c \quad \text{and} \quad x_2 \quad \text{is} \quad A_{i2}^c \quad \text{and} \ldots \text{and} \quad x_1 \quad \text{is} \quad A_{iN}^c \quad \text{then} \quad u(k) = -g_i x(k) \] (38)

where \( i = \{1, \ldots, N\} \), \( N \) is the number of fuzzy rules which is the number of faults to represented, \( \{x_1, \ldots, x_1\} \) are current states of the plant, \( A_{ij}^c \) are the gaussians membership functions like:

\[ A_{ij}^c = \exp \left( -\frac{(x_j - c_{ij})^2}{\sigma_{ij}^2} \right) \] (39)

where \( c_{ij}^c \) and \( \sigma_{ij}^c \) are constants to be tuned. Furthermore, \( I_i^c \) represents the related control gain.

Similar to fuzzy system plant, fuzzy control representation is integrated as:

\[ w_i = \prod_{j=1}^{l} A_{ij}^c (x_j) \] (40)

and

\[ u(k) = \frac{\sum_{i=1}^{N} w_i (g, x(k))}{\sum_{i=1}^{N} w_i} \] (41)

The configuration of FLC is integrated to the already explored plant where final representation is given as closed loop system of linear feedback plant as shown in Fig. 12.

\[ x(k+1) = \frac{\sum_{i=1,j=1}^{N} h_i w_j \left( a_i - c_i l_j B_i^p x(k) + B_i^p g, ref \right)}{\sum_{i=1,j=1}^{N} h_i w_j} \] (42)

where ref is the reference to be followed by controller and the variables \( i \) and \( j \) are used due to fuzzy rules.
interconnections as the representation of different linear plants and respective controllers

From this representation, stability needs to be stated as the following Lyapunov function:

\[
V(x(k)) = x^T(k)Px(k)
\]  

and

\[
\Delta V(x(k)) = v(x(k + 1)) - v(x(k))
\]  

where

\[
V(k + 1) = x(k + 1)^TPx(k + 1)
\]

\[
V(k + 1) = \left( \sum_{i=1,j=1}^{N} h_{wj} \left( a_i - c_i g_j B_j^p \right) x(k + 1) + B_j^p g_j ref \right)^T P
\]

\[
\sum_{i=1,j=1}^{N} h_{wj}
\]

Therefore

\[
\Delta V(x(k)) = \left( \sum_{i=1,j=1}^{N} h_{wj} \left( a_i - c_i g_j B_j^p \right) x(k + 1) + B_j^p g_j ref \right)^T P
\]

\[
\sum_{i=1,j=1}^{N} h_{wj}
\]

and

\[
\Delta V(x(k)) = \left( \sum_{i=1,j=1}^{N} h_{wj} \left( a_i - c_i g_j B_j^p \right) x(k + 1) + B_j^p g_j ref \right)^T P
\]

\[
\sum_{i=1,j=1}^{N} h_{wj}
\]
then by considering \( \text{ref=0} \)

\[
\Delta V(x(k)) = \sum_{i=1, j=1}^{N} (h_{ij}w_{ij}) \begin{bmatrix} - \left( a_{i} P_{c, c_j} B_{ji} \right)^{T} & \left( c_{i} g_{j} B_{ji} \right)^{T} \end{bmatrix} P_{a_{i}} + a_{i} P_{a_{i}} + \left( c_{i} g_{j} B_{ji} \right) P_{i} \left( c_{i} g_{j} B_{ji} \right) x - xPr \]

\[
\sum_{i=1, j=1}^{N} (h_{ij}w_{ij})^{T}
\]

(48)

It is important to remember that

\[
\Delta V(x(k)) \leq 0
\]

(49)

and

\[
\left( a_{i} P_{c, c_j} B_{ji} \right)^{T} \left( c_{i} g_{j} B_{ji} \right) P_{a_{i}} + a_{i} P_{a_{i}} + \left( c_{i} g_{j} B_{ji} \right) P_{i} \left( c_{i} g_{j} B_{ji} \right) < 0
\]

(50)

where \( P > 0 \), \( \| \cdot \| \) is the Euclidean norm and it is possible to define

\[
g_{j} > \left\| B_{ji} \right\|
\]

(51)

This condition has to be given for every single time delay and local fault appearance. Furthermore the stability and the convergence of states should be assured by the adequate selection of matrices \( l \) and the related parameters from both fuzzy systems. In this case a recommendable procedure to follow is multi-objective optimization in order to define those suitable values.

6. Conclusions

Present approach shows the integration of two techniques in order to perform reconfiguration. These two approaches are followed, in cascade mode, structural reconfiguration and control reconfiguration. Although there is no formal verification in order to follow this sequence, it has been adopted since structural reconfiguration provides settle conditions for control reconfiguration. The use of a real-time scheduling algorithm in order to approve or disapprove modifications on computer network behaviour allows time delays bounding during a specific time window. This local time delay bounding allows the design of a control law capable to cope with these new conditions.

As has been shown in this work, fuzzy logic control based upon Takagi Sugeno or other structures allows the possibility of control reconfiguration as long as linear models based representation of the plant are available. Despite of local faults and bounded time delays appearance, several conditions should be fulfill in order to be able to follow this proposal, for instance, plant should be observable and controllable during the whole nonlinear behaviour as well as the states should be present during undesirable situations.

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